1.6 Classify Polygons

Before You classified angles.
Now You will classify polygons.
Why? So you can find lengths in a floor plan, as in Ex. 32.

Key Vocabulary
- **polygon**
  side, vertex
- **convex**
- **concave**
- ***n*-gon**
- **equilateral**
- **equiangular**
- **regular**

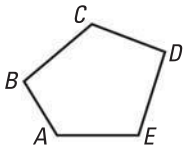
KEY CONCEPT *For Your Notebook*

Identifying Polygons

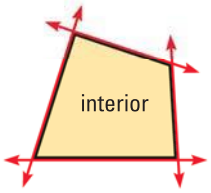
In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

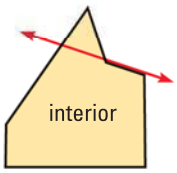
Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is *vertices*. A polygon can be named by listing the vertices in consecutive order. For example, *ABCDE* and *CDEAB* are both correct names for the polygon at the right.



A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called *nonconvex* or **concave**.



convex polygon

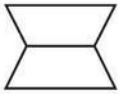


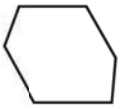
concave polygon

EXAMPLE 1 Identify polygons

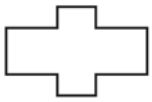
READ VOCABULARY
A *plane figure* is two-dimensional. Later, you will study three-dimensional *space figures* such as prisms and cylinders.

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

a. 

b. 

c. 

d. 

Solution

a. Some segments intersect more than two segments, so it is not a polygon.

b. The figure is a convex polygon.

c. Part of the figure is not a segment, so it is not a polygon.

d. The figure is a concave polygon.

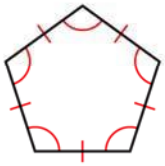
CLASSIFYING POLYGONS A polygon is named by the number of its sides.

Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon

Number of sides	Type of polygon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

The term ***n*-gon**, where *n* is the number of a polygon's sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent. In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.

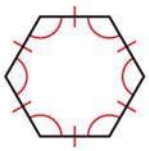


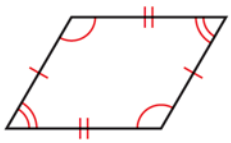
regular pentagon

EXAMPLE 2 Classify polygons

READ DIAGRAMS
Double marks are used in part (b) of Example 2 to show that more than one pair of sides are congruent and more than one pair of angles are congruent.

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

a. 

b. 

c. 

Solution

a. The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.

b. The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.

c. The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

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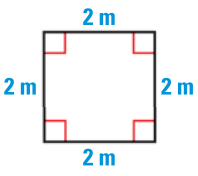
GUIDED PRACTICE for Examples 1 and 2

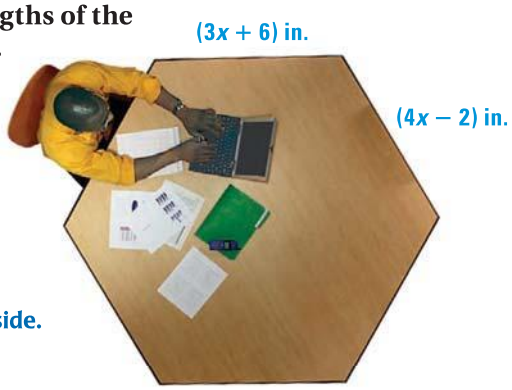
1. Sketch an example of a convex heptagon and an example of a concave heptagon.
2. Classify the polygon shown at the right by the number of sides. *Explain* how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.



EXAMPLE 3 Find side lengths

READ VOCABULARY
Hexagonal means "shaped like a hexagon."

ALGEBRA **A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.**



Solution

First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

$3x + 6 = 4x - 2$ **Write equation.**

$6 = x - 2$ **Subtract $3x$ from each side.**

$8 = x$ **Add 2 to each side.**

Then find a side length. Evaluate one of the expressions when $x = 8$.

$3x + 6 = 3(8) + 6 = 30$

▸ The length of a side of the table is 30 inches.

GUIDED PRACTICE for Example 3

3. The expressions $8y°$ and $(9y - 15)°$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.

1.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 37, 39, and 40

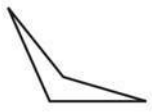
SKILL PRACTICE

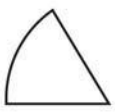
1. VOCABULARY *Explain* what is meant by the term n-gon.

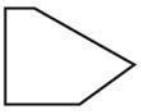
2. ★ WRITING Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain.*

EXAMPLE 1
on p. 42
for Exs. 3–7

IDENTIFYING POLYGONS **Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*.**

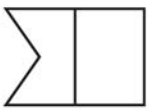
3. 

4. 

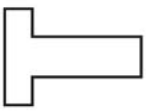
5. 

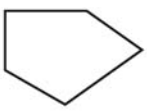
6. 

7. ★ MULTIPLE CHOICE Which of the figures is a concave polygon?

(A) 

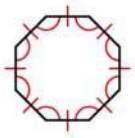
(B) 

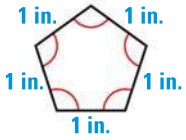
(C) 

(D) 

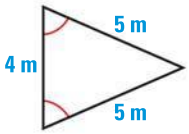
EXAMPLE 2
on p. 43
for Exs. 8–14

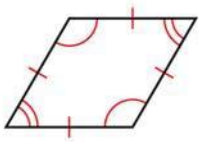
CLASSIFYING **Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. *Explain* your reasoning.**

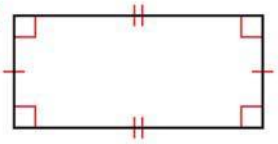
8. 

9. 

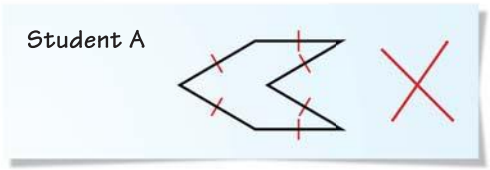
10. 

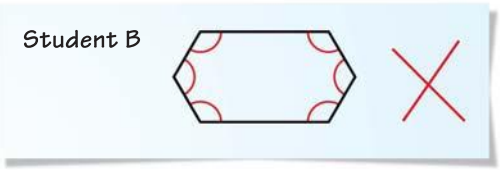
11. 

12. 

13. 

14. **ERROR ANALYSIS** Two students were asked to draw a regular hexagon, as shown below. *Describe* the error made by each student.





EXAMPLE 3
on p. 44
for Exs. 15–17

15. **ALGEBRA** The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. **ALGEBRA** The expressions $(9x + 5)^\circ$ and $(11x - 25)^\circ$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. **ALGEBRA** The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES **Tell whether the statement is *always*, *sometimes*, or *never* true.**

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING **Draw a figure that fits the description.**

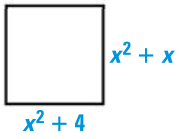
24. A triangle that is not regular

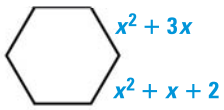
25. A concave quadrilateral

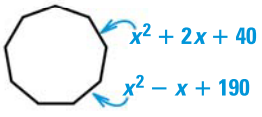
26. A pentagon that is equilateral but not equiangular

27. An octagon that is equiangular but not equilateral

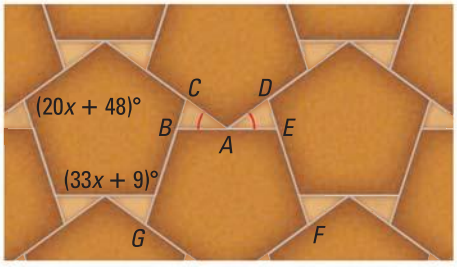
ALGEBRA **Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of x.**

28. 

29. 

30. 

31. CHALLENGE Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.



PROBLEM SOLVING

32. ARCHITECTURE Longwood House, shown in the photograph on page 42, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.

a. Tell whether the red polygon in the diagram is *convex* or *concave*.

b. Classify the red polygon and tell whether it appears to be regular.

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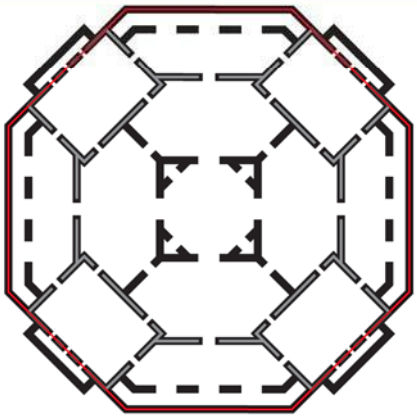

EXAMPLE 2 on p. 43 for Exs. 33–36

SIGNS **Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral*, *equiangular*, or *regular*.**

33.

34.

35.

36.

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37. ★ MULTIPLE CHOICE Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, -4)$. Which of the following could be the other two vertices?

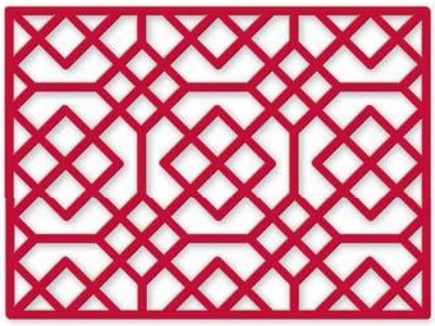
Ⓐ $C(4, 4)$ and $D(4, -4)$ Ⓑ $C(-4, 4)$ and $D(-4, -4)$

Ⓒ $C(8, -4)$ and $D(8, 4)$ Ⓓ $C(0, 8)$ and $D(0, -8)$

38. MULTI-STEP PROBLEM The diagram shows the design of a lattice made in China in 1850.

a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.

b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.

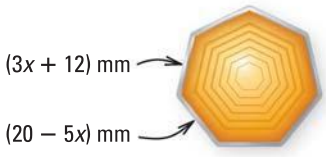


○ = WORKED-OUT SOLUTIONS on p. WS1

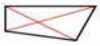
★ = STANDARDIZED TEST PRACTICE

EXAMPLE 3 on p. 44 for Ex. 39

39. ★ **SHORT RESPONSE** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain*.



40. ★ **EXTENDED RESPONSE** A segment that joins two nonconsecutive vertices of a polygon is called a *diagonal*. For example, a quadrilateral has two diagonals, as shown below.

Type of polygon	Diagram	Number of sides	Number of diagonals
Quadrilateral		4	2
Pentagon	?	?	?
Hexagon	?	?	?
Heptagon	?	?	?

a. Copy and complete the table. *Describe* any patterns you see.

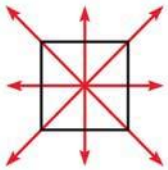
b. How many diagonals does an octagon have? a nonagon? *Explain*.

c. The expression $\frac{n(n-3)}{2}$ can be used to find the number of diagonals in an n-gon. Find the number of diagonals in a 60-gon.

41. **LINE SYMMETRY** A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

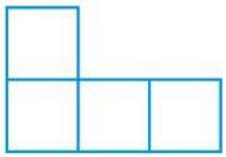
a. A regular triangle **b.** A regular pentagon

c. A regular hexagon **d.** A regular octagon



regular quadrilateral
4 lines of symmetry

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge.



MIXED REVIEW

PREVIEW Prepare for Lesson 1.7 in Exs. 43–51.

Solve the equation.

43. $\frac{1}{2}(35)b = 140$ *(p. 875)* **44.** $x^2 = 144$ *(p. 882)* **45.** $3.14r^2 = 314$ *(p. 882)*

Copy and complete the statement. *(p. 886)*

46. 500 m = __?__ cm **47.** 12 mi = __?__ ft **48.** 672 in. = __?__ yd

49. 1200 km = __?__ m **50.** $4\frac{1}{2}$ ft = __?__ yd **51.** 3800 m = __?__ km

Find the distance between the two points. *(p. 15)*

52. $D(-13, 13)$, $E(0, -12)$ **53.** $F(-9, -8)$, $G(-9, 7)$ **54.** $H(10, 5)$, $J(-2, -2)$